Image Conjecture

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I. Introduction

This formula is used to calculate the number of images formed by two mirrors when kept at some angle α.

A. Explanation

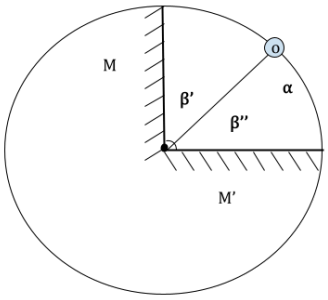

Fig. 1: Sample Diagram

Let the angle between mirrors M and M' be α. Angle between mirror M and object O be β'. Angle between mirror M' and object O be β''. Then the number of images formed will be

$$n = \left\lfloor \pi \frac{\sin \alpha}{\beta} \right\rfloor \tag{1}$$

where $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x and all angles are taken in radians. Conditions for β:

- If $\beta' > \beta''$ then β' will be taken as β in the formula.
- If $\beta'' > \beta'$ then β'' will be taken as β in the formula.
- If $\beta' = \beta''$ then any can be taken as β in the formula.

II. Examples [1]

1) Find the total number of images formed if two plane mirrors are inclined at an angle $\frac{\pi}{2}$ and object is situated at an angle of $\frac{\pi}{6}$ from one of them.

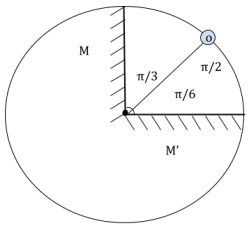

Fig. 2: Solution 1

As angle between mirror M and O is greater than angle between M' and O.Then $\alpha = \frac{\pi}{2}$ and $\beta = \frac{\pi}{3}$.

$$n = \left\lfloor \pi \frac{\sin \alpha}{\beta} \right\rfloor = \left\lfloor \pi \frac{\sin\left(\frac{\pi}{2}\right)}{\frac{\pi}{3}} \right\rfloor = 3 \tag{2}$$

Number of images formed will be 3.

2) Find the total number of images formed if two plane mirrors are inclined at an angle $\frac{\pi}{3}$ and object is situated at an angle of $\frac{\pi}{6}$ from one of them.

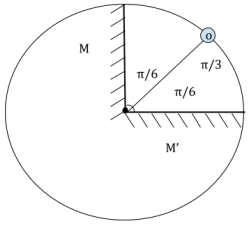

Fig. 3: Solution 2

As angle between mirror M and O is equal to angle between M' and O.Then $\alpha = \frac{\pi}{3}$ and $\beta = \frac{\pi}{3}$.

$$n = \left\lfloor \pi \frac{\sin \alpha}{\beta} \right\rfloor = \left\lfloor \pi \frac{\sin\left(\frac{\pi}{3}\right)}{\frac{\pi}{3}} \right\rfloor = \lfloor 5.1961 \rfloor = 5 \tag{3}$$

Number of images formed will be 5.

References

[1] M. C. P. Limited, “Physics and Its Dynamics,” *Geometrical Optics*, vol. 5, pp. 2–3, 2022.